Thinker™ EDUCATION
MATH SPECIALISTS
PRIMARY & SECONDARY

2024

S3 ADDITIONAL MATH WA1 MATH (EXPRESS)

KRANJI SECONDARY SCHOOL

DETAILED SOLUTIONS

Detailed solutions are crafted following the methods taught at Thinker Education and are offered as a guiding reference. Any logically sound mathematical answers are accepted.

For Thinker parents, the respective levels' blank question papers and detailed solutions have been uploaded to Teams.

For others, please Whatsapp us at 9831 9770 to obtain the question papers for your child to practise.

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KRANJI SECONDARY SCHOOL

WEIGHTED ASSESSMENT 1 2024

ADDITIONAL MATHEMATICS 4049

Level	:	Secondary Three	**Date**	:	______ Feb 2024
Stream	:	Express	**Duration**	:	45 minutes
Name	:	DETAILED SOLUTIONS ( )	**Marks**	:	/ 30
Class	:	Secondary ________			

READ THESE INSTRUCTIONS FIRST:

Do not open this question paper until you are told to do so.

Write your name, class and register number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

Give non-exact numerical answers correct to three significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 30.

Setter: Ms Lee

This question paper consists of 8 printed pages, including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

Quadratic Equations & Inequalities

1 **(i)** Solve the equation $2x+7=\frac{9}{x}$. [2]

$x(2x+7)=9$

$2x^2+7x-9=0$

$(2x+9)(x-1)=0$

$2x+9=0$ or $x-1=0$

$2x=-9$ $\quad x=1$ #

$x=-\frac{9}{2}$ #

(ii) Hence, find the solution(s) of the equation $2y+7\sqrt{y}-9=0$. [2]

$2(\sqrt{y})^2+7(\sqrt{y})-9=0$

Let $\sqrt{y}$ be x.

$2x^2+7x-9=0$

$\therefore x=-\frac{9}{2}$ or $x=1$

$\sqrt{y}=-\frac{9}{2}$ $\quad \sqrt{y}=1$

$y=(-\frac{9}{2})^2$ $\quad y=(1)^2$

$y=\frac{81}{4}$ $\quad y=1$ #

(reject)

Surds

2 Simplify $\sqrt{150}+\sqrt{54}-\sqrt{96}$ **without using a calculator.** [2]

$= \sqrt{6\times 25}+\sqrt{6\times 9}-\sqrt{6\times 16}$

$= 5\sqrt{6}+3\sqrt{6}-4\sqrt{6}$

$= 4\sqrt{6}$

#

Surds

3 A cuboid of volume $(18+11\sqrt{2})\text{ cm}^3$ stands on its square base. If the area of the base is $(3+2\sqrt{2})\text{ cm}^2$, find, **without using a calculator**, the length of its height in the form $(p+q\sqrt{2})$ cm, where p and q are integers. [3]

Base Area x height = volume

$(3+2\sqrt{2})(h) = 18+11\sqrt{2}$

$h = \frac{18+11\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$

$= \frac{54-36\sqrt{2}+33\sqrt{2}-22(2)}{(3)^2-(2\sqrt{2})^2}$

$= 10-3\sqrt{2}$

$\therefore$ height $= (10-3\sqrt{2})$ cm

#

Surds

4 Given that $2a+10\sqrt{3}-15-a\sqrt{3}=b\sqrt{3}-9$, find the values of the integers a and b. [2] 2

$2a-15+10\sqrt{3}-a\sqrt{3} = -9+b\sqrt{3}$

By comparing coefficients,

$2a-15=-9$ $\quad$ $10-a=b$

$2a=6$ $\quad$ $10-3=b$

$a=3$ $\quad$ $b=7$

\# \#

Surds

5 Solve $x-4=\sqrt{7+2x}$. [3]

$(x-4)^2 = (\sqrt{7+2x})^2$

$x^2-8x+16 = 7+2x$

$x^2-10x+9=0$

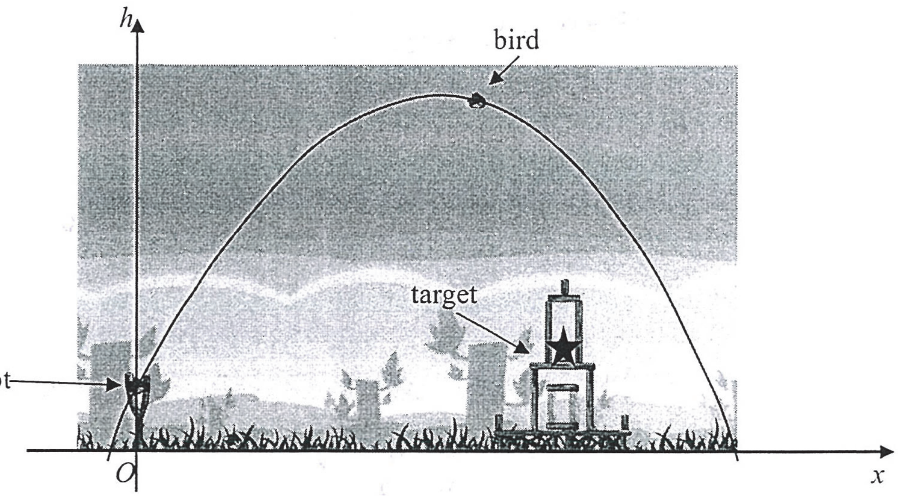
$(x-9)(x-1)=0$

$x-9=0$ or $x-1=0$

$x=9$ $\quad$ $x=1$

\# (reject)

6 The picture below is taken from a mobile game where a bird is launched from a slingshot to hit a target. Its height, h m, above the ground can be modelled by the equation $h = -0.1x^2 + 4x + 1.8$, where x m is the horizontal distance from the slingshot.



(a) Find the height of the bird above the ground when it just left the slingshot. [1]

when $x=0$,

$h = 1.8$

$\therefore$ height $= 1.8$ m #

(b) Find the greatest height of the bird after it was launched from a slingshot. [2]

$h = -0.1(x^2 - 40x) + 1.8$

$= -0.1\left[x^2 - 40x + \left(\frac{40}{2}\right)^2 - \left(\frac{40}{2}\right)^2\right] + 1.8$

$= -0.1\left[(x-20)^2 - 400\right] + 1.8$

$= -0.1(x-20)^2 + 40 + 1.8$

$= -0.1(x-20)^2 + 41.8$

$\therefore$ greatest height $= 41.8$ m #

(c) Find the horizontal distance travelled by the bird when it is first at 12.9m above the ground. [3]

when $h = 12.9$,

$-0.1(x-20)^2 + 41.8 = 12.9$

$-0.1(x-20)^2 = -28.9$

$(x-20)^2 = 289$

$x - 20 = \pm\sqrt{289}$

$x - 20 = 17$ or -17

$x = 37$ or 3

$\therefore$ horizontal distance $= 3$ m #

Quadratic Equations & Inequalities

7 **(i)** Show that the equation $2qx^2 + px = 2q - p$ has real roots for all real values of p and q. [4]

$2qx^2 + px - 2q + p = 0$

discriminant $= (p)^2 - 4(2q)(-2q+p)$

$= p^2 + 16q^2 - 8pq$

$= p^2 - 8pq + 16q^2$

$= (p)^2 - 2(p)(4q) + (4q)^2$

$= (p-4q)^2$

since $(p-4q)^2 \geq 0$ for all real values of p and q, the equation has real roots. (shown)

#

(ii) Hence find the relationship between p and q for which the equation $2qx^2 + px = 2q - p$ has equal roots. [1]

For equal roots,

$(p-4q)^2 = 0$

$p - 4q = 0$

$p = 4q$ #

Quadratic Equations & Ineaualities

8 The curve $y = ax^2 + bx + 1$ has a minimum point and no intersection with the x-axis. State the conditions that must apply to the constants a and b. [3]

$a > 0$ and $b^2 - 4ac < 0$

$(b)^2 - 4(a)(1) < 0$

$b^2 - 4a < 0$

$\therefore\ a > 0,\ b^2 - 4a < 0$ #

Quadratic Equations & Ineaualities

9 Find the range of values of p for which $-2x^2 + 5x - 3 - p$ is always negative. [2]

For $-2x^2 + 5x - 3 - p < 0$,

$b^2 - 4ac < 0$

$(5)^2 - 4(-2)(-3-p) < 0$

$25 - 24 - 8p < 0$

$1 - 8p < 0$

$-8p < -1$

$p > \frac{1}{8}$ #

End Of Paper