MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\cdots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\triangle = \frac{1}{2} bc \sin A$$

MATHEMATICAL FORMULAE

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface of a cone = πrl

Surface area of sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{mean} = \frac{\sum fx}{\sum f}$$

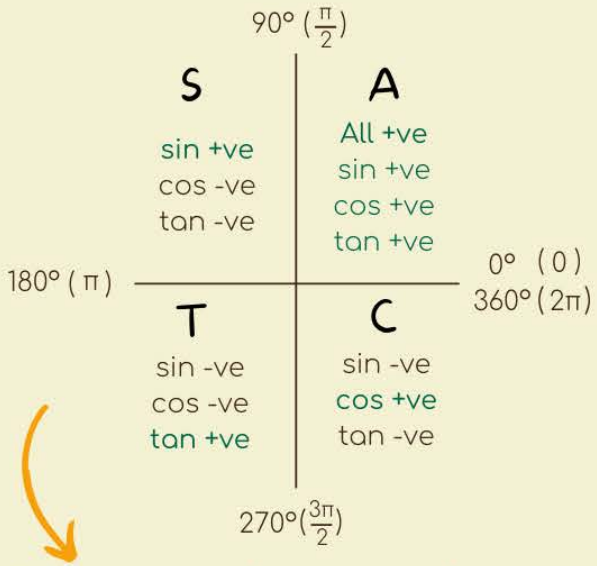
$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Trigonometry

Radian & Degree

Degree	0°	30°	45°	60°	90°	180°	270°	360°
Radian	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

ASTC



	sin α	cos α	tan α
A	sin α	cos α	tan α
S	sin (180° - α)	-cos (180° - α)	-tan (180° - α)
T	- sin (180° + α)	-cos (180° + α)	tan (180° + α)
C	- sin (360° - α)	cos (360° - α)	-tan (360° - α)

Supplementary Angles

$\sin(180^\circ - \theta) = \sin\theta$

$\cos(180^\circ - \theta) = -\cos\theta$

$\tan(180^\circ - \theta) = -\tan\theta$

Complementary Angles

$\sin(90^\circ - \theta) = \cos\theta$

$\cos(90^\circ - \theta) = \sin\theta$

$\tan(90^\circ - \theta) = \cot\theta$

Special Angles

	0 0°	$\frac{\pi}{6}$ 30°	$\frac{\pi}{4}$ 45°	$\frac{\pi}{3}$ 60°	$\frac{\pi}{2}$ 90°
sin θ	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
cos θ	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

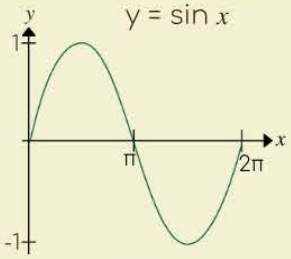
Negative Angles

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

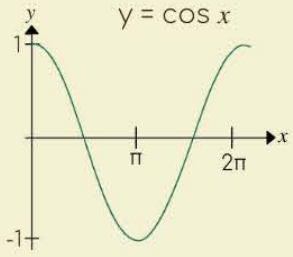
Trigonometry Graphs



$y = a \sin bx$

amplitude = a

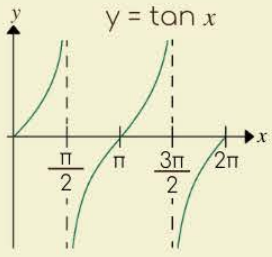
period = $\frac{2\pi}{b}$



$y = a \cos bx$

amplitude = a

period = $\frac{2\pi}{b}$



$y = a \tan bx$

amplitude = undefined

period = $\frac{\pi}{b}$

Principal Values

$-90^\circ \le \sin^{-1} x \le 90^\circ$
$(-1 \le x \le 1)$

$0^\circ \le \cos^{-1} x \le 180^\circ$
$(-1 \le x \le 1)$

$-90^\circ \le \tan^{-1} x \le 90^\circ$
(x is a real number)

Simple Identities

$\sin^2 A + \cos^2 A = 1$

$\tan^2 A + 1 = \sec^2 A$

$\cot^2 A + 1 = \operatorname{cosec}^2 A$

Addition Formula

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Double Angle Formula

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$
$= 2\cos^2 A - 1$
$= 1 - 2\sin^2 A$

$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

R – Formula

$a \sin\theta \pm b \cos\theta = R \sin(\theta \pm \alpha)$

$a \cos\theta \pm b \sin\theta = R \cos(\theta \mp \alpha)$

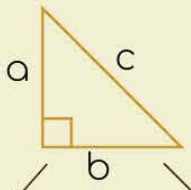
$R = \sqrt{a^2 + b^2}$ and $\tan\alpha = \frac{b}{a}$

Trigonometry

Techniques for Triangles

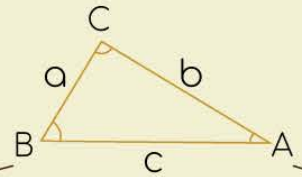
Triangles

- Right - Angled Triangles



  - Given: sides & angles → TOA CAH SOH
    - $\tan\theta = \frac{O}{A}$
    - $\cos\theta = \frac{A}{H}$
    - $\sin\theta = \frac{O}{H}$
    - $\cot\theta = \frac{1}{\tan\theta}$
    - $\sec\theta = \frac{1}{\cos\theta}$
    - $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$
    - $\tan\theta = \frac{\sin\theta}{\cos\theta}$
    - $\cot\theta = \frac{\cos\theta}{\sin\theta}$
  - Given: only sides → Pythagoras Theorem $c^2 = a^2 + b^2$

- Non Right - Angled Triangles



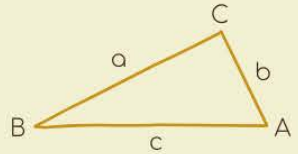
  - Given: 1 corresponding side & 2 angles → Sine Rule
    - For finding sides $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
    - For finding angles $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
  - Given: 1 corresponding side
    - For finding angles $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
  - Given: 2 sides & 1 included angle → Cosine Rule
    - For finding sides $a^2 = b^2 + c^2 - 2bc\cos A$
  - Given: 3 sides
    - For finding angles $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Obtuse Angles

$\sin\theta = \sin(180° - \theta)$

$\cos\theta = -\cos(180° - \theta)$

Area of Triangles



Area of $\triangle ABC = \frac{1}{2}ab\sin C$

Radian Measure

- π radians $= 180°$
- $180° = \pi$ radians
- 1 radians $= \frac{180°}{\pi}$
- $1° = \frac{\pi}{180}$ radians

MENSURATION FORMULAE

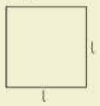
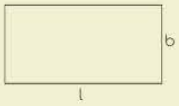
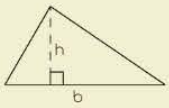
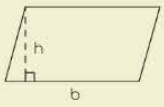
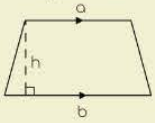
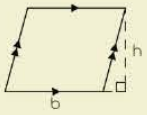
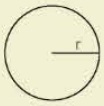
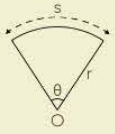
Conversion of units

Length
1 cm = 10 mm
1 m = 100 cm
1 km = 1000 m

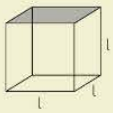
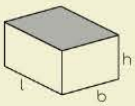
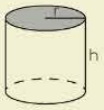
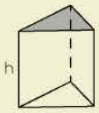
Area
$1\ m^2 = 100\ cm \times 100\ cm = 10\ 000\ cm^2$
$1\ km^2 = 1000\ m \times 1000\ m = 1\ 000\ 000\ m^2$

Volume
$1\ cm^3 = 1\ m\ell$
$1\ \ell = 1000\ m\ell = 1000\ cm^3$
$1\ m^3 = 100\ cm \times 100\ cm \times 100\ cm = 1\ 000\ 000\ cm^3$

Perimeter and Area of Plane Figure

Plane Figures	Area	Perimeter
Square 	l^2	$4l$
Rectangle 	$l \times b$	$2(l + b)$
Triangle 	$\frac{1}{2} \times b \times h$	
Parallelogram 	$b \times h$	
Trapezium 	$\frac{1}{2}(a + b)\ h$	
Rhombus 	$b \times h$	
Circle 	πr^2	$2\pi r$
Sector 	$\frac{\theta}{360^o} \times \pi r^2$ (where θ is in degrees) or $\frac{1}{2} r^2 \theta$ (where θ is in radians)	Arc length $= \frac{\theta}{360^o} \times 2\pi r$ (where θ is in degrees) or $= r\theta$ (where θ is in radians)

Volume and Surface Area of Solids

Solids	Volume	Total Surface Area
Cube 	l^3	$6l^2$
Cuboid 	$l \times b \times h$	$2(lb + lh + bh)$
Cylinder 	$\pi r^2 h$	$2\pi rh + 2\pi r$
Prism 	Area of Cross Section x h	Perimeter of the cross section x height + 2(base area)
Pyramid 	$\frac{1}{3}$ x base area x h	base area + triangular lateral faces
Cone 	$\frac{1}{3}\pi r^2 h$	Bass + curved surface area
Sphere 	$\frac{4}{3}\pi r^3$	$4\pi r^2$
Hemisphere 	$\frac{2}{3}\pi r^3$	curved surface area + circle